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O'RAIFEARTAIGH'S THEOREM FOR THE GROUPS CONTAINING SPINOR GENERATORS

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A b s t r a c t

O'Raifeartaigh's theorem is extended to the groups which contain spinor generators.

O'RAIFEARTAIGH'S THEOREM FOR THE GROUPS CONTAINING SPINOR GENERATORS

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It is well known that the main obstacle on the way of nontrivial combining of Lorentz invariance and internal symmetry is the O'Raifeartaigh's theorem [1]. From this theorem follows that in the irreducible representations of the finite Lie groups the mass operator P^2 has either only one fixed value or a continuous spectrum of values.

Recently a number of papers have appeared in which the groups of symmetry differ from Lie groups - so called groups of supersymmetry have been discussed [2 - 5]. The supersymmetry groups contain transformations with the generators which are spinors with respect to Lorentz group. Therefore, for keeping the right connection between spin and statistics we must introduce the anticommutators between the spinor generators. Since the permutation relations for the groups containing spinor generators contain also the anticommutators, the hope have appeared to avoid O'Raifeartaigh's theorem [4 - 6].

In the present note we will show that O'Raifeartaigh's theorem can be extended to any finite group, which contains the spinor generators.

Let us consider a group with an algebra G, containing the Poincare algebra (with the generators $J_{\mu\nu}$ and

P_ς ($\mu,\nu,\varsigma = 0,1,2,3$) as a subalgebra. Let $Q_{\alpha i}$ denote the operators of the algebra G , which transform as the fourcomponent spinors, where α (α=1,2,3,4) is the spinor index, i - a set of indexes for exemple, of internal group of symmetry, and let A_i denote the rest operators of the algebra G (excepting the operators $J_{\mu\nu}$ and P_ς).

Theorem. If the mass operator $P^2 = P_\mu P_\mu$ and every finite power thereof are self-adjoint on the Hilbert space H on which any representation of G operates, and if there exists a discrete point m^2 in the spectrum of P^2 on H, then the eigenspace H_m belonging to the point m^2 is closed, and is invariant with respect to the operators representing the algebra G .

That H_m is closed is proved in the same way as in [1]. Let us show that the space H_m is invariant with respect to the operators representing the algebra G .

Let us consider the operators A_i . Since the permutation relationsof the operators A_i with themselvs and P_μ contain only the commutators the proof of Lemma I of the paper [1] is valid. Then, analogous to [1] we conclude that the space H_m is invariant with respect to the operators representing A_i .

The statement of Lemma I [1] is valid for the spinor operators $Q_{\alpha i}$ too. Indeed, in view of Lorentz invariance the commutation relations of the operators P_μ and $Q_{\alpha i}$ have a form

$$[P_\mu, Q_{\alpha i}] = a(V_\mu Q)_{\alpha i}, \qquad (1)$$

where V_μ is four x four matrixs and a - an arbitrary number (in particular, zero). Taking into account that $[P_\mu, P_\nu]=0$, from the Yacobi identity for the operators P_μ and $Q_{\alpha i}$ we obtain

$$[P_\mu,[P_\nu, Q_{\alpha i}]] + [P_\nu,[Q_{\alpha i}, P_\mu]] = 0. \qquad (2)$$

From (1(and (2) follow that $V_\mu V_\nu = 0 (\mu,\nu = 0,1,2,3)$. As a result

$$[P_\mu,[P_\nu, Q_{\alpha i}]] = 0 \qquad (3)$$

The equality (3) is the statement of Lemma I of the paper [1] ($n=2$). Then, using Lemma II of the paper [1] analogous to [1] we conclude that the space H_m is invariant with respect to the operators representing $Q_{\alpha i}$.

Thus, the space H_m is invariant with respect to the whole algebra G. Q.E.D.

From the generalized O'Raifeartaigh's theorem follow that in the irreducible representations of any finite group both containing spinor generators and without them, the mass operator P^2 has either only one fixed value or a continuous spectrum of values.

Thus, the including of spinor operators into the symmetry groups doesn't give us the possibility to explain the mass differences within the multiplets of the ordinary internal symmetry groups.

R e f e r e n c e s

1. L.O'Raifeartaigh, Phys. Rev. Lett., 14, 575 (1965).
2. Yu.A.Golfand, E.P.Lichtman, Zh.Exp.Teor.Fiz., Pizma Red., 13, 452 (1971).
3. J.Wess, B.Zumino, Nucl.Phys., B70, 39 (1974).
4. J.Wess, B.Zumino, Phys.Lett., B49, 52 (1974).
5. A.Salam, J.Strathdee, Nucl.Phys., B76, 477 (1974).
6. B.Zumino, CERN preprint TH-1901 (1974).

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