И Н С Т И Т У Т
ЯДЕРНОЙ ФИЗИКИ СОАН СССР

ПРЕПРИНТ И Я Ф 74 - 48

AHARONOV-BOHM EFFECT AND THE PRINCIPLE OF LOCALITY IN QUANTUM MECHANICS

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Новосибирск

1974

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The principle of locality of interactions in the quantum theory is discussed in connection with the interpretation of Aharonov-Bohm effect. In the frame of nonrelativistic quantum mechanics this principle is formulated in terms of Heisenberg equations of motion for the coordinate and velocity operators. These equations contain field strengths only and do not contain potentials explicitly. It can be obtained from them that the force influences the motion of wave packet when the distance from the centre of a packet to the force region is less than the mean square dimension of the packet even if the wave function vanishes in this region.

1. In 1959 Aharonov and Bohm /1/ drew attention of physicists to the peculiar role of electromagnetic potentials in the quantum mechanics. For example they demonstrated that solenoid placed beyond a screen with two slits changed the interference pattern for charged particles in spite of vanishing of wave function in the magnetic field region. On the other hand the vector potential which directly enter Schrödinger equation does not vanish outside the solenoid. On these grounds the authors come to the conclusion that the principle of locality of interactions cannot be expressed in terms of fields alone and the use of the vector potential has physical significance. The question raised by Aharonov and Bohm was discussed afterwards in various aspects by many authors (see in particular the reviews /2,3/ and refernces therein).

In the present article we wish to discuss the form in which the concept of local action of field strengths can be retained in the quantum mechanics. Let us emphasize that we discuss only the interpretation of Aharonov-Bohm effect its existence seeming to us unquestionable both from the theoretical and experimental /4/ point of view.

2. In classical mechanics the locality means that the particle acceleration at a given point is determined by the magnitudes of field strengths at the same point. The conception of a field itself is due to the possibility of local measurement by means of test pointlike particles.

When proceeding to the quantum description the classical relation between acceleration and field strengths is replaced by the analogous relation between operators. This relation remains local and does not contained potentials. However its consequences prove to be not so simple and obvious as in classical physics due to the presence of nonlocalized states in the quantum mechanics.

The experiments proposed by Aharonov and Bohm are the examples of such situation. Consider in particular a particle motion in the field of a long thin solenoid. The localized wave packet represents the quantum-mechanical analogue of classical pointlike particle. It is easy to show that if the packet localized near certain point at initial moment does not touch the solenoid region during its motion, the density of probability does not depend on the presence of magnetic field in the solenoid. Let us emphasize that in the region where the packet moves only magnetic field vanishes while the vector potential is not

generally speaking equal to zero.

However the situation changes drastically if the initial packet consists of two parts passing both sides of the solenoid. In such case the interference of packet parts depends on the presence of field in solenoid in spite of exponential vanishing of the wave function in the magnetic field region. It seems paradoxical that magnetic field influences on a motion of particles which do not appear in the field region.

Thus the time development of the state is not determined by field strengths at the points where the wave function is nonvanishing. It means that the conception of locality cannot be formulated for ψ-functions without use of the vector potential /1/ (see also ref./5/).

At the same time the conception of locality of Heisenberg equations remains true, of course. However as it is clear from the discussed examples it implies that the field affects the packet motion if it does not vanish inside the region occupied by a packet as a whole. In particular for a packet consisting of two parts its dimension along a given direction should be defined as a corresponding mean square distance from the centre of the packet.

3. To clarify the statements above consider for example the time development of distribution $w(v_y, t)$ of velocity v_y (the packet moves along x-axis, z-axis is directed along the solenoid axis). This distribution is given by the matrix element of operator $\hat{K} = \delta(\hat{v}_y(t) - v_y)$ where $\hat{v}_y(t)$ is the Heisenberg operator of velocity.

Representing the operator $\hat{K}$ in the form

$$\hat{K} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\lambda\, e^{-i\lambda v_y} e^{i\lambda \hat{v}_y(t)} \tag{1}$$

we obtain after simple transformations its time derivative

$$i\frac{d\hat{K}}{dt} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\lambda\, e^{-i\lambda v_y}\int_0^{\lambda} d\lambda'\, e^{i\lambda'\hat{v}_y}\frac{d\hat{v}_y}{dt} e^{-i\lambda'\hat{v}_y + i\lambda\hat{v}_y} \tag{2}$$

Since the operator $d\hat{v}_y/dt$ is proportional to the operator of force, the distribution $w(v_y, t)$ varies only if the field strengths do not vanish.

Now we substitute into equ.(2) the explicit expression for the operator of force acting on the particle in magnetic field and use commutation relations for coordinate and velocity operators. Then

$$i\frac{d\hat{K}}{dt} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\lambda\, e^{-i\lambda v_y}\int_0^{\lambda} d\lambda' \frac{e}{2mc}\Big\{\hat{v}_x + \frac{e\hbar}{m^2c}\int_0^{\lambda'} d\lambda''\, H(\hat{x}, \hat{y} + \frac{\hbar}{m}\lambda''), H(\hat{x}, \hat{y} + \frac{\hbar}{m}\lambda')\Big\} e^{i\lambda\hat{v}_y} \tag{3}$$

In this expression the braces designate the anticommutator.

With use of equ.(3) we can now express $dw(v_y, t)/dt$ through the coordinate wave function $\psi(x, y; t)$ taken at the time moment t. It is convenient to choose the gauge where

$$A_x = -\int_{-\infty}^{x} dy'\, H(x, y'), \quad A_y = 0 \tag{4}$$

Then

$$i\frac{dw(v_y, t)}{dt} = i\langle\psi|\frac{d\hat{K}}{dt}|\psi\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy\, e^{(i/\hbar) m v_y y} \times \int_{-\infty}^{+\infty} dy'\, e^{-(i/\hbar) m v_y y'}\int_{-\infty}^{+\infty} dx\, \psi^*(x, y; t)\int_y^{y'} d\eta \frac{e}{2m^2c} \times \Big\{-i\hbar\frac{\partial}{\partial x} + \frac{e}{c}\int_{-\infty}^{\eta} d\eta'\, H(x, \eta'), H(x, \eta)\Big\}\psi(x, y'; t) \tag{5}$$

It is clear from this formula that the packet motion depends on the magnitudes of magnetic field taken not only at points where the wave function does not vanish but on the lines connecting such points. Therefore, as it was noted above, for the motion of the packet consisting of two parts not only the size of this parts is essential but the distance between them as well.

Let us emphasize that the above statements are to the point for the case of potential force for which the formula analogous to equ.(5) can be obtained

4. Now we summerize the conclusions. If one tries to formulate the principle of locality directly for wave functions one must ascribe the physical significance to potentials. Such point of view seems however inconsistent since there are no ways of local measurement of potentials. On the other hand there is the consistent way to state the principle of locality in terms of Heisenberg equations of motion.

From this principle it results for ψ-functions that an interaction takes place if the distance from the packet centre to the force region is less than the mean square dimension of the packet in the direction perpendicular to its velocity. Such understanding of locality accords well to the general conceptions of quantum mechanics.

Note that the generalization of statements to relativistic case meets essential difficulties connected with the lack of detailed Heisenberg formalism for the operators of coordinates and velocities.

In conclusion we wish to thank S.A.Kheifets and I.B.Khriplovich for prolonged and fruitful discussions.

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Ответственный за выпуск С.Н.РОДИОНОВ
Подписано к печати I.УП.74г., МН09387
Усл. 0,4 печ.л., тираж 200 экз.Бесплатно
Заказ № 48