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И Н С Т И Т У Т
ЯДЕРНОЙ ФИЗИКИ СОАН СССР

ПРЕПРИНТ И Я Ф 74-4

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THE WAVE TAILS AND NONSPHERICAL PERTURBATIONS OF RELATIVISTIC GRAVITATIONAL COLLAPSE

Новосибирск

1974

Submitted to Lett. Nuovo cim. 6. 12. 1973.

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A b s t r a c t

The behaviour of a wave signales which propagated in the Schwarzshild space are investigated. The interaction of a wave with the curvature of the space-time results in the power tails $(t-r)^{-\ell-1}$ after the main pulse of waves.

We consider the wave propagation in the space with the line element

$$ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (1)$$

(We use units in which $c=1$, $G=1$). For scalar, electromagnetic and gravitational fields after some procedures (see [1-4]) one have to solve the equation

$$\psi'' + [k^2 - U_\ell(x)]\psi = 0, \quad x = r + 2M\ln\left(\frac{r}{2M}-1\right) \qquad (2)$$

where k is the frequence and $U_\ell(x)$ is "potential" which depends on the nature of the field and on the partial harmonic number ℓ. Their asymptotic forms are

$$U_\ell(x \gg 2M) \simeq \frac{\ell(\ell+1)}{x^2}\left(1+\frac{4M}{x}\ln\frac{x}{2M}\right) \qquad (3)$$

$$U_\ell(-x \gg 2M) \simeq Const\cdot\exp(x/2M) \qquad (4)$$

The Schroedinger-type equation (2) is well known in the Quantum Scattering theory (see [5]). The first term in (3) is the centrifugal barrier, and the term

$$U^{(st)} = \frac{4M\ell(\ell+1)}{x^3}\ln\frac{x}{2M} \qquad (5)$$

is the slowly decreasing "scattering potential".

We introduce the function

$$\Psi_k(x) = A_k(x)\left[e^{ikx} + B_k(x)e^{-ikx}\right] \qquad (6)$$

$$A_k(-\infty) = 1 \;, \quad B_k(+\infty) = 0 \qquad (7)$$

which is the outgoing-wave-solution of the equation (2). This solution was found in our paper [3]. In the Scattering theory the power-law decrease of the scattering potential $\mathcal{U}^{(st)}$ results in the anomalous scattering [5]. The related effect of the anomalous tunneling occurs for the treated case too. For $|2Mk| \ll 1$ one has

$$\Psi_k(x) = C_1(2Mk)^{\ell+1}\left[1 + C_2(2Mk)\ln(2Mk)\right]e^{ikx} + \ldots, \quad |kx| \gg 1 \qquad (8)$$

$$= C_3 x^{-\ell}(2Mk) + \ldots + C_4(x/2M)^{\ell+1}(2Mk)^{2\ell+3}\ln(2Mk) + \ldots \qquad (9)$$

$$|kx| \ll 1$$

The formulae (8),(9) work out the problem of the wave tails (see [6]). A wave packet is described by the integral

$$\Psi(t,x) = \int_{-\infty}^{\infty} f_k \Psi_k(x) e^{-ikt}\,dk \qquad (10)$$

The spectral function f_k depends on the initial properties of the packet. The logarithmic terms in

$\Psi_k(x)$ lead to the large contribution from the small k region for $t \gg 2M$. Assume that a wave packet was emitted out of the region $2M < r < r_0$, the characteristic length time of the packet beeng τ_0. The signal detected at the distance $r \gg r_0$ has the following form.

At first the field amplitude has the quick change with the characteristic time τ_0 . Then it fall's off according to

$$\Psi \simeq \frac{Const}{\tau^{\ell+1}} \;, \quad \tau_0 \ll \tau = t - r \ll r \qquad (11)$$

For $\tau \simeq t \gg r$ the asymptotic law is

$$\Psi \simeq \frac{Const \cdot r^{\ell+1}}{t^{2\ell+2}} \qquad (12)$$

It points out that the power tail (11) appertains to the wave field and the tail (12) appertains to the quasi-static field.

In reality the space-time near the gravitating mass is the Schwarzshild one only for $r \le R_0$ therefore the power laws (11) and (12) are valid only for $\tau \ll R_0$.

The importont problem which is solved by the formilae (8)-(12) is the problem of the behaviour of nonspherical perturbations of the metric (1) during gravitational collapse.

As it was shown elsewhere [7] a collapsing star emits a wave with the exponential tail with characteristic time $\tau_0 = (4M)^{-1}$. The evolution of this perturbations at large time is described by the laws (11) and (12). The law (12) was obtained in the paper [4] for this case.

The appearance of the power tails is the direct effect of the wave signal transfer through curved space. Exactly the same tails appear when a wave packett propagate near the rotating star. It is readily shown by the like method.

We are grateful to A.G.Doroshkevich, Ia.B.Zeldovich and I.D.Novikov for useful discussions.

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Ответственный за выпуск С.Н.РОДИОНОВ
Подписано к печати 13.Ш.74 г. МН 08128
Усл. 0,2 печ.л., тираж 200 экз. Бесплатно.
Заказ № 4 .

Отпечатано на ротапринте в ИЯФ СО АН СССР.