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B.N.Breizman, D.D.Ryutov

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PENETRATION OF A LONGITUDINAL MAGNETIC FIELD INTO A PLASMA OF A LINEAR DISCHARGE

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In experiment /1/ the propagation time of a magnetic field pulse from the boundary to the axis of an electrode ended plasma colunm was measured. An interesting feature ofthe experiment was that the mean signal velocity turned to be anomalously high (at least an order of magnitude higher than Alfven velocity). In this paper one of the possible mechanisms of such a fast penetration of a field into a plasma is discussed.

It is assumed that a plasma fills a cylindrical volume ($0 \leq r \leq R$; $-L/2 \leq z \leq L/2$) between two perfectly conducting electrodes. The constant longitudinal magnetic field is denoted by $\vec{H}_0$. Besides that let's suppose that at a plasma boundary a pulsed magnetic field $|\vec{H}| \ll |\vec{H}_0|$ of a form

$$\vec{H}(R,z,t) = \vec{H}_{max}(z) \begin{cases} 0 & ; \quad t<0 \\ \sin \frac{\pi t}{\tau} & ; \quad 0<t<\tau \\ 0 & ; \quad t>\tau \end{cases} \qquad (1)$$

is created by an external coil ($r \simeq R$; $-\ell/2 \leq z \leq \ell/2$). The pulse duration τ is considered to be much greater than the inverse cyclotron frequency ω_{H_0} ($\omega_{H_0}\tau \gg 1$).

First, consider the case of a small amplitude pulse, when plasma currents do not produce any instability and a usual Ohm's law is valid:

$$j_z = \frac{\omega_0^2}{4\pi\nu} E_z ;$$
$$\vec{j}_\perp = -\frac{\omega_0^2}{4\pi\omega_{H_0}} \frac{[\vec{E}\times\vec{H}_0]}{|\vec{H}_0|} , \qquad (2)$$

where $\omega_0 \equiv (4\pi n e^2/m)^{1/2}$ is an electron plasma frequency, ν - electron - ion collision frequency. Only an electron contribution in current is taken into account, because in the experiment under consideration the pulse velocity is much greater than Alfven velocity, and ion current is negligibly small.

Substituting relations (2) into a quasistationary Maxwell equations and taking into account that $(c/\omega_0)(\nu\tau)^{1/2} \ll R$ one gets:

$$\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right) H_z = -\frac{\omega_0^4}{\omega_{H_0}^2 c^4}\frac{\partial^2}{\partial t^2} H_z \qquad (3)$$

The boundary conditions for H_z at $z = \pm L/2$ are:

$$H_z(r, \pm L/2) = 0$$

Equation (3) is a wave equation for helicons in plasma. The radial velocity of the basic harmonic ($H_z \backsim \cos \pi z/L$) is given by

$$V = \frac{\pi}{L}\frac{\omega_{H_0} c^2}{\omega_0^2} \qquad (4)$$

Higher harmonics penetrate into a plasma faster than the basic one, but for a long exciting coil ($\ell \sim L$) their amplitudes are much less than that of the basic one (the latter point comes immediatly from the Fourier expantion of a magnetic field on the plasma boundary). Therefore, the penetration velocity in the case $\ell \sim L$ is determined by the formula (4) namely. Strictly speaking this formula is valid only at $|\vec{H}_{max}| \ll |\vec{H}_0|$. But one can also use it as an estimate at $|\vec{H}_{max}| \sim |\vec{H}_0|$ and even at $|\vec{H}_{max}| \gg |\vec{H}_0|$ (in the latter case the cyclotron frequency to be calculated for a field $\vec{H}_{max}$). If the coil becomes shorter, then V increases and tends to its maximum value V^* as $\ell/R \to 0$:

$$V^* \sim \frac{1}{R}\frac{\omega_{H_0} c^2}{\omega_0^2}$$

Note that in the case of a short coil ($l \ll L$) the penetration time is insensitive to the boundary conditions (the presence or the absence of electrodes). The estimation (4) is consistent with the experimental velocity values at a plasma density of $n \sim 2\,10^{12} cm^{-3}$. But the dependence $v \propto n^{-1}$ differs from an experimental one ($v \propto n^{-1/2}$). This discrepancy may be due to the fact that the pulse amplitude in experiment /1/ was not sufficiently small. To check experimentally whether the propagation of helicon is really cause of the penetration of magnetic field into plasma one should measure the φ - component ofthis field (φ - component to be obviously present in the helicon).

The high amplitude pulse may give rise to current instability in plasma and as a consequence to anomalous resistivity. If so, the current will be "frozen" at some critical level which depends on plasma parameters and corresponds to the marginal stability. In helicon z and φ - components of a current are of the same order of magnitude. Since in all existing models of anomalous resistivity the threshold of instability for current perpendicular to the magnetic field is less than that for the parallel one, we shall consider that the instability and anomalous resistivity are caused by the current j_φ . In order to estimate the penetration time for the turbulent regime, we shall assume the anomalous resistivity to be connected with a sound type instability and take the following interpolation for the critical current dependence on the electron temperature

$$j_\varphi = Ane\left(\frac{T}{m}\right)^{1/2}\left[\left(\frac{m}{M}\right)^{1/4} + \left(\frac{T_0}{T}\right)^{\alpha}\right] \qquad (5)$$

Here T_0 is an initial electron temperature, $A>0$ and $\alpha>0$ - some constants, A being of the order of unity. This formula for

$T = T_0$ gives the critical current for isothermal ($T_i = T_e$) plasma $j_\varphi \sim ne(T_0/m)^{1/2}$. Moreover, for high temperature formula (5) gives asymptotical dependence $j_\varphi = Ane(m/M)^{1/4}(T/m)^{1/2}$, obtained in the paper /2/.

Denoting the characteristic penetration depth by $\Delta(t)$ one can write that

$$\frac{H_z(R,t)}{\Delta(t)} \sim \frac{4\pi}{c} j_\varphi$$

$$\frac{E_\varphi}{\Delta(t)} \sim \frac{1}{c}\frac{\partial}{\partial t} H_z(R,t) \qquad (6)$$

$$n\frac{\partial T}{\partial t} \sim j_\varphi E_\varphi$$

By means of these relations it is possible to express a temperature inside a skin in terms of a magnetic field perturbation at the plasma boundary:

$$T \sim H_z^2(R,t)/8\pi n \qquad (7)$$

From the expressions (5 - 7) it comes that

$$\Delta \sim \frac{c}{\omega_0 A}\left[\left(\frac{m}{M}\right)^{1/4} + \left(\frac{8\pi n T_0}{H_z^2}\right)^{\alpha}\right]^{-1} \qquad (8)$$

Introducing the explicit time dependence of H_z at $t < \tau$ (cf.(1)) and taking $\Delta = R$, one finds a penetration time in the case of the anomalous resistivity existence:

$$t_0 \sim \tau\left(\frac{8\pi n T_0}{\pi H_{z\,max}^2}\right)^{1/2}\left(\frac{AR\omega_0}{c}\right)^{1/2\alpha} \qquad (9)$$

This formula is valid only if $R < \frac{c}{\omega_0 A}\left(\frac{M}{m}\right)^{1/4}$, because in the opposite case the maximum penetration depth is smaller than R . The dependence of t_0 on H_z coincides with experimental one. As for the dependence on concentration, it is determined by a choice of α (in particular, at $\alpha \gg 1$ one gets $t_0 \backsim n^{1/2}$).

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