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A b s t r a c t.

Relations for the induced pseudoscalar coupling constant in μ capture on proton and the amplitude of pion electroproduction near the threshold are formulated which are expected to be valid with percent accuracy.

Recently a large number of relations have been obtained based on the hypothesis of partial conservation of hadron axial current /1,2/ a_μ

$$\partial_\mu a_\mu = \frac{\mu^2}{c}\varphi \qquad (1)$$

where φ is the renormalized π- meson field operator, μ - the meson mass, C - constant.

Apart from relation (1) assumption is used that only one (in most cases) or two /3/ terms in expansion of amplitudes in series of μ/m_{int}, m_{int} being some internal mass, may be kept /4/. It is assumed that m_{int} is large as compared with μ.

But already from the comparison of the Goldberger-Treiman relation /5/ with experimental data we learn that m_{int} is not so large. The deviation from the Goldberger-Treiman relation is due to quadratic in μ/m_{int} terms and equal to 13% that implies $m_{int} \sim 3\mu$. This violation of the Goldberger-Treiman relation results in 13% uncertainty in the value of constant C which enter many theoretical predictions. For example the πN-

scattering lengths prove to be proportional to c^2.

In this note we use the hypotheses a) on the possibility of expanding amplitudes in series of μ/m_{int} and b) axial current conservation in the limit of * $\mu^2 \to 0$ to derive two relations which are expected to be valid with a few percents accuracy. The first one expresses the induced pseudoscalar constant g_P in μ capture on proton in terms of the radius of the nucleon axial formfactor. In obtaining this relation we neglect terms $(\mu/m_{int})^4$. The second prediction is valid up to terms $(\mu/m_{int})^2$ and relates the amplitude of electroproduction of pions on the threshold to that of photoproduction and the radii of the pion electromagnetic and nucleon axial formfactors. In deriving

* Let us note that just this version of relation (1) is used to obtain most of the known so far results, the only exceptions being the calculation of $\pi\pi$-scattering lengths [4] and decay [6] $\eta \to 3\pi$.

the relations we essentially use results obtained in papers [1] and [7].

Because of the high accuracy of the relations obtained experimental check of them might serve as a crucial test of the hypotheses a) and b) and is very desirable although seems difficult at present time.

The matrix element of the axial current between proton and neutron states is of the form

$$M_\mu = \langle p | a^+_\mu(0) | n \rangle = g(k^2)\bar{u}_2 \gamma_\mu \gamma_5 u_1 - h(k^2) k_\mu \bar{u}_2 \hat{k} \gamma_5 u_1 \qquad (2)$$

where u_1, u_2 are wave functions of the initial and final nucleons with 4momenta p_1 and p_2, $k = p_1 - p_2$, $g(k^2)$, $h(k^2)$ are the axial and pseudoscalar formfactors

At small k^2 the pole contribution dominates the formfactor $h(k^2)$ and we write it out explicitly

$$h(k^2) = \frac{f_\pi g_{\pi NN}}{\sqrt{2}\, m_N} \frac{1}{k^2 - \mu^2} + \varepsilon(k^2) \qquad (3)$$

where $\frac{g^2_{\pi NN}}{4\pi}=14.6$, f_π is the $\pi\to\mu\nu$ decay constant, $f_\pi = 0.93$. It follows from the assumption b) above that /1/

$$g(k^2,\mu^2=0)=k^2 h(k^2,\mu^2=0) \tag{4}$$

Expanding $g(k^2)$, $h(k^2)$ in series in k^2 we get

$$g(k^2=0,\mu^2=0)=\frac{f_\pi g_{\pi NN}}{\sqrt{2}\, m_N} \tag{5}$$

$$\frac{\partial g}{\partial k^2}(k^2=0,\mu^2=0)=\tau(k^2=0,\mu^2=0) \tag{6}$$

Equation (5) being the Golberger-Treiman relation.

Because of the nonvanishing pion mass the analogous relations for the physical quantities at $\mu^2\neq 0$ are only approximate. We assume that violations of both relations (5) and (6) are of the same order, i.e. 13%. But as the contribution of $\tau(k^2)$ to $h(k^2)$ is small as compared with the pole contribution relation (6) allows to predict $h(k^2)$ with a few percents accuracy. In particularly for g_P we get

$$g_P=2m_N m_\mu h=-8.85+2m_N m_\mu \frac{\partial g}{\partial k^2}(0) \tag{7}$$

Different analyses /8/ of the existing data on the neutrino experiment give $dg(k^2)/dk^2(k^2=0) = (0.4 - 1.2\ \mathrm{Gev})^{-2}$. The corresponding correction to the pole value of g_P is (1.5 - 14)%. Experimentally g_P is measured with 40% accuracy, $g_P = -(10.3 \pm 4.4)$. /9/

Let us consider now the amplitude of electroproduction of π^+- meson on proton at small, of order μ, electron momentum transfer. The matrix element of this process is written as

$$T=\frac{4\pi\alpha}{k^2}\bar{v}_2\gamma_\lambda v_1\,\bar{u}_2 M_\lambda u_1 \tag{8}$$

where v_1, v_2 are the wave functions of the initial and final electrons, u_1, u_2 of proton and neutron with momenta p_1, p_2 respectively.

Keeping only quadratic in the pion momentum q and the virtual photon momentum k terms we get for the pole contribution

$$M_\lambda^{pole}=-i\sqrt{2}\,f(q^2)\Big\{\hat{q}\gamma_5\frac{1}{\hat{p}_2+\hat{q}-m_N}\Big[\gamma_\lambda(1+F_p'(0)k^2)-\frac{\kappa^p}{2m_N}\sigma_{\lambda\xi}k_\xi\Big]+\Big[\gamma_\lambda F_n'(0)k^2-\frac{\kappa^n}{2m_N}\sigma_{\lambda\xi}k_\xi\Big].$$

$$\cdot \frac{1}{\hat{p}_1 - \hat{q} - m_N} \hat{q}\gamma_5 \} - i\sqrt{2}\, f(\mu^2) \frac{(2q-k)_\lambda}{(q-k)^2 - \mu^2} \cdot \tag{9}$$

$$\cdot (\hat{q} - \hat{k})\gamma_5$$

where $F_p'(0)$, $F_n'(0)$, $F_\pi'(0)$ are the first derivatives of the proton, neutron and pion charge formfactors in respect of k^2 at $k^2=0$, $f(q^2)$ is the vertex function of the pseudovector πNN coupling. At $q^2=\mu^2$ $f(\mu^2)$ coincides with PS-PV coupling constant $f(\mu^2) = \frac{g_{\pi NN}}{2m}$ while at $q^2 \neq \mu^2$ $f(q^2)$ is related to the formfactors $g(k^2)$, $h(k^2)$

$$g(q^2) - q^2 h(q^2) = -\frac{\sqrt{2} f(q^2)}{c} \frac{\mu^2}{q^2 - \mu^2} \tag{10}$$

Longitudal part of M_λ is equal to [3]

$$k_\lambda M_\lambda = -i\sqrt{2}\, f((k-q)^2) \frac{q^2-\mu^2}{(q-k)^2 - \mu^2} (\hat{q} - \hat{k})\gamma_5 \tag{11}$$

If M_λ is written as

$$M_\lambda = M_\lambda^{pole} + \Delta M_\lambda + M_\lambda^{\perp} \tag{12}$$

where $k_\lambda M_\lambda^{\perp} = 0$, it follows from equation (11) that ΔM_λ may be taken as

$$\Delta M_\lambda = i\sqrt{2}\, f(\mu^2) F_\pi'(0) \frac{k_\lambda(2kq - k^2)}{(q-k)^2 - \mu^2} (\hat{q} - \hat{k})\gamma_5 - \tag{13}$$

$$- i\sqrt{2}\, f(q^2)\gamma_\lambda\gamma_5 + i\sqrt{2}\, f(\mu^2)[F_p'(0) - F_n'(0)] k_\lambda \hat{q}\gamma_5$$

The transverse part of M_λ may be represented as a sum of six invariant amplitudes

$$M_\lambda^{\perp} = \sum_{s=1}^{6} V_s(\nu, kq, k^2) O_{s\lambda}, \quad \nu = \frac{(k+q)(p_1+p_2)}{4m_N}$$

where we choose the six independent invariants as follows

$$O_{1\lambda} = \gamma_5 \sigma_{\lambda\xi} k_\xi$$

$$O_{2\lambda} = \frac{1}{2m_N} \gamma_5 (\hat{q} - \hat{k})[(p_1+p_2)_\lambda (qk) - q_\lambda (k(p_1+p_2))]$$

$$O_{3\lambda} = \gamma_5 [\gamma_\lambda (qk) - q_\lambda \hat{k}] \tag{14}$$

$$O_{4\lambda} = -i\epsilon_{\lambda\gamma\rho\sigma}\gamma_\gamma k_\rho q_\sigma$$

$$O_{5\lambda} = \frac{1}{2m_N} \gamma_5 (\hat{q} - \hat{k})[k_\lambda (kq) - q_\lambda k^2]$$

$$O_{6\lambda} = \gamma_5 [k_\lambda \hat{k} - \gamma_\lambda k^2]$$

Using relation (1) and assumption on equal time commutator of electromagnetic current and axial charge one can obtain /7/

$$V_1(0,0,0)=0, \quad V_6(0,0,0)=ic\left.\frac{\partial g(\kappa^2)}{\partial \kappa^2}\right|_{\kappa^2=0} \tag{15}$$

Finally we get for the amplitude of pion electroproduction neglecting cubic and higher order in k,q terms

$$\begin{aligned} M_\lambda = &-if\sqrt{2}\Big\{\hat{q}\gamma_5\frac{1}{\hat{p}_2+\hat{q}-m_N}\Big[\gamma_\lambda\big(1+F_p'(0)\kappa^2\big)- \\ &-\frac{\kappa^p}{2m_N}\sigma_{\lambda\xi}\kappa_\xi\Big]+\Big[\gamma_\lambda F_n'(0)\kappa^2-\frac{\kappa^n}{2m_N}\sigma_{\lambda\xi}\kappa_\xi\Big]\frac{1}{\hat{p}_1-\hat{q}-m_N}\hat{q}\gamma_5+ \\ &+(\hat{q}-\hat{\kappa})\gamma_5\frac{(2q-\kappa)_\lambda+2F_\pi'(0)\big(q_\lambda\kappa^2-\kappa_\lambda(\kappa q)\big)}{(q-\kappa)^2-\mu^2}- \\ &-\gamma_\lambda\gamma_5-\big[F_p'(0)-F_n'(0)\big]\kappa_\lambda\hat{q}\gamma_5\Big\}+icg(0)\gamma_5(\kappa_\lambda\hat{\kappa}-\gamma_\lambda\kappa^2)- \\ &-i\epsilon_{\lambda\gamma\rho\sigma}\gamma_\gamma\kappa_\rho q_\sigma V_4(0,0,0)+\gamma_5\big(\gamma_\lambda(q\kappa)-q_\lambda\hat{\kappa}\big)V_3(0,0,0)+\gamma_5\sigma_{\lambda\xi}\kappa_\xi\,\nu\frac{\partial V_1}{\partial\nu}(0,0,0) \end{aligned} \tag{16}$$

where the magnitudes of V_3, V_4, $dV_1/d\nu$ may be determined from experiments on pion photoproduction We have not considered the contribution of the

isobar N*(1236) and therefore relation (16) is valid for the threshold value of the amplitude. Formfactor V_4 may be omitted in this limit.

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